AP Calculus AB

Area Between Two Curves

1) $x = y^3 \quad x = y^2$

$$\text{Area} = \int_0^1 [y^2 - y^3]\,dy$$

$$\left[\tfrac{1}{3}y^3 - \tfrac{1}{4}y^4 + C\right]\Big|_0^1$$

$$\tfrac{1}{3} - \tfrac{1}{4} = \boxed{\tfrac{1}{12}}$$

2)

$$\text{Area} = \int_0^1 \left[12y^2 - 12y^3 - (2y^2 - 2y)\right]dy$$

$$= \int_0^1 \left[-12y^3 + 10y^2 + 2y\right]dy$$

$$= \left[-3y^4 + \tfrac{10}{3}y^3 + y^2 + C\right]\Big|_0^1$$

$$= -3 + \tfrac{10}{3} + 1 = \boxed{\tfrac{4}{3}}$$

3)

$$\text{Area} = 2\int_0^2 \left[2x^2 - (x^4 - 2x^2)\right]dx$$

$$= 2\int_0^2 \left[4x^2 - x^4\right]dx$$

$$= 2\left[\tfrac{4}{3}x^3 - \tfrac{1}{5}x^5 + C\right]\Big|_0^2$$

$$= \boxed{2\left[\tfrac{32}{3} - \tfrac{32}{5}\right]}$$

4)

$$\text{Area} = 2\int_0^1 \left[x^2 - (-2x^4)\right]dx$$

$$= 2\int_0^1 \left[x^2 + 2x^4\right]dx$$

$$= 2\left[\tfrac{1}{3}x^3 + \tfrac{2}{5}x^5 + C\right]\Big|_0^1$$

$$= \boxed{2\left[\tfrac{1}{3} + \tfrac{2}{5}\right]}$$

5) $y = x^2\ ; \quad y = 2 - x$

$$\text{Area} = \int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx$$

$$= \tfrac{1}{3}x^3\Big|_0^1 + \left(2x - \tfrac{1}{2}x^2\right)\Big|_1^2$$

$$= \tfrac{1}{3} + (4-2) - \left(2 - \tfrac{1}{2}\right)$$

$$= \boxed{\tfrac{5}{6}}$$

OR

$y = x^2 \qquad x + y = 2$

$x = \sqrt{y} \qquad x = 2 - y$

$$\text{Area} = \int_0^1 \left[2 - y - y^{1/2}\right]dy$$

$$= \left[2y - \tfrac{1}{2}y^2 - \tfrac{2}{3}y^{3/2} + C\right]\Big|_0^1$$

$$= \left[2 - \tfrac{1}{2} - \tfrac{2}{3}\right] = \boxed{\tfrac{5}{6}}$$

6)

$$\text{Area} = \int_{-2}^{-1}\left[-x + 2 - (4 - x^2)\right]dx + \int_{-1}^{2}\left[4 - x^2 - (-x+2)\right]dx + \int_{2}^{3}\left[-x + 2 - (4 - x^2)\right]dx$$

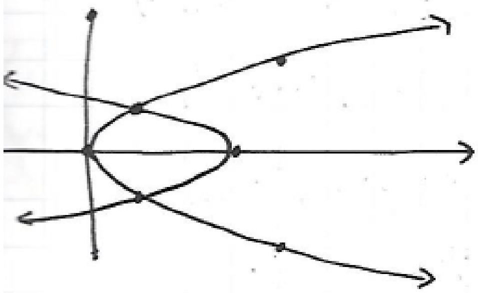
$$= \int_{-2}^{-1}\left[x^2 - x - 2\right]dx + \int_{-1}^{2}\left[2 + x - x^2\right]dx + \int_{2}^{3}\left[x^2 - x - 2\right]dx$$

$$= \left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 2x\right]\Big|_{-2}^{-1} + \left[2x + \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]\Big|_{-1}^{2} + \left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 2x\right]\Big|_{2}^{3}$$

$$= \left[-\tfrac{1}{3} - \tfrac{1}{2} + 2\right] - \left[-\tfrac{8}{3} - 2 + 4\right] + \left[4 + 2 - \tfrac{8}{3}\right] - \left[-2 + \tfrac{1}{2} + \tfrac{1}{3}\right] + \left[9 - \tfrac{9}{2} - 6\right] - \left[\tfrac{8}{3} - 2 - 4\right]$$

7) $x - y^2 = 0 \qquad x + 2y^2 = 3$

$x = y^2 \qquad x = 3 - 2y^2$



y-values of intersection: $y = -1$ & $y = 1$

$\int_{-1}^{1} [3 - 2y^2 - y^2] dy$

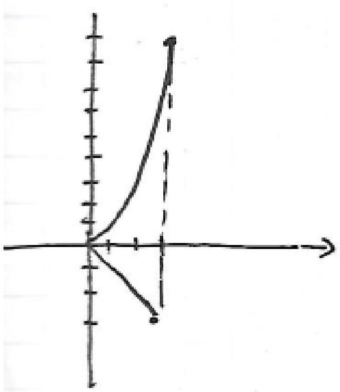
$\int_{-1}^{1} [3 - 3y^2] dy$

$[3y - y^3 + C] \Big|_{-1}^{1}$

$[3 - 1] - [-3 + 1]$

$\boxed{4}$

8) $y = x^2 \qquad y = -x$



$\text{Area} = \int_0^3 [x^2 - (-x)] dx$

$= \int_0^3 [x^2 + x] dx$

$[\frac{1}{3}x^3 + \frac{1}{2}x^2 + C] \Big|_0^3$

$9 + \frac{9}{2} = \boxed{\frac{27}{2}}$